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Besides the above named periodicals, there are now issued quite a number of Educational periodicals, each having a Math'l Department, and, last, but not least, the "Yates County Chronicle," a weekly paper, having a Mathematical Department under the direction of Dr. Samuel H. Wright."

Prof. N. R. Leonard, of Iowa State University at Iowa City, writes—"A very large and brilliant fire-ball passed this place at 10–30 yesterday evening (Feb. 12th)—its size apparently half that of the moon—its course slightly north of west, and marked by a brilliant and broad train of light and by three separate explosions, and followed at an interval of three minutes by a report that some compared to a fusilade of musketry, others to the rumbling of a train of cars.

SOLUTIONS OF PROBLEMS IN NUMBER 1, *VOL. II.*

Solutions of problems in No. 1 have been received as follows:

From J. M. Arnold, 52; R. W. Ryan, 52; Prof. A. B. Evans, 54, 55 & 58; Edgar Frisby, 56; E. S. Farrow, 51, 52, 53, 54, 55, 56, & 57; J. M. Greenwood, 54 & 56; Henry Gunder, 52, 56 & 58; G. W. Hill, 55 & 56; Prof. E. W. Hyde, 58; H. Heaton, 52, 53, 56, 57 & 58; Phil. Hoglan, 52; Prof. W. W. Johnson, 58; Artemas Martin, 52 & 56; O. D. Oathout 52 & 56; E. B. Seitz, 52, 54, 56, 57 & 58; Walter Siverly, 52, 54 & 56 Werner Stille, 54 & 56.

The following credits, due, were omitted, by an oversight, in the January No. Prof. J. Scheffer, 41, 42, 43, 44, 47 & 49; E. B. Seitz, 41, 43, 44, 45 & 48; L. Regan, 41, 42, 43 & 45; August Zielinski, 43 & 45. Also, Th. L. De Land, in addition to the solutions for which he was credited in the Jan. No., sent a correct and very elegant solution of 48.

It must not be inferred that, when solutions are not selected for publication, they are thought *unworthy*. On the contrary, many solutions that are not published are among the very best that we receive, and are passed, in making our selections, because our space will not permit us to introduce the details which they contain. For instance, we have on hand very elegant solutions of No. 48, by Prof. Hyde, Prof. Evans and Prof. Johnson, but, at this time, our space will not permit us to publish them. Although very brief, it is believed that the published solution of 58, and also that of 54, will be found sufficient to guide the student in making out a solution in detail if he desires it.

51.—"Given $(x^2 + y^2)y = 39, \ldots. (1)\ x^4 + y^4 = 97, \ldots\ldots (2)$ to find x and y by quadratics."

SOLUTION BY CADET EDWARD S. FARROW, WEST POINT, N. Y.

Put $y = ax$. Substituting this value in (1) and (2), multiplying (1) by x and (2) by a, and adding, we shall have

$$(a^5 - a^3)x^4 + 39x = 97a. \ldots. (3)$$

Applying Mr. Hill's method for the solution of biquadratic equations, we can easily find from (3) that $x = \frac{4}{3}a$; $\therefore y = ax = \frac{3}{4}x^2$, or $x^2 = \frac{4}{3}y$. Substituting this in (2), we have $y^4 + \frac{16}{9}y^2 = 97$: whence $y = 3$. The corresponding value of x is 2.

52.—"A hare starts 10 rods north of a hound and runs at an angle of 45° with the meridian. The hound pursues in a straight line and in such a direction that he will intercept the hare without changing his course. Supposing the hound to run n times as fast as the hare, how far will he run before he catches the hare?"

SOLUTION BY O. D. OATHOUT, READ, IOWA.

Put 10 rods $= a$, and $45° = \theta$. Let $x =$ the distance ran by the hare, then $nx =$ distance ran by the hound.

By Geometry, we have $n^2x^2 = a^2 + x^2 + 2ax\cos\theta$,

or $$(n^2 - 1)x^2 - 2a\cos\theta . x = a^2;$$

whence $$2(n^2 - 1)x - 2a\cos\theta = \pm\sqrt{[4a^2\cos^2\theta + 4a^2(n^2 - 1)]}.$$

$$\therefore \quad x = \frac{a\cos\theta \pm \sqrt{(a^2\cos^2\theta + a^2n^2 - a^2)}}{n^2 - 1}.$$

Substituting the values of a and θ and reducing, we get

$$x = \frac{5\sqrt{2}.[1 \pm \sqrt{(2n^2 - 1)}]}{n^2 - 1}. \quad \therefore nx = \frac{5n\sqrt{2}.[1 \pm \sqrt{(2n^2 - 1)}]}{n^2 - 1}.$$

53.—If a ball of 6 inches diameter is discharged from a canon at the rate of one mile in 7 seconds, how much greater force will be required to throw a ball of double the weight with the same velocity, taking into account the resistance of the air and the diameters of the balls?"

SOLUTION BY EDWARD S. FARROW.

If we suppose the balls of the same material, or density constant, the diameter of the 2nd ball will be $6\sqrt[3]{2}$.

The actual resistance of the atmosphere at any time, is composed of two terms, the one due to the inertia of the displaced particles, the other to the difference of the atmospheric pressure, in front and rear. Both terms must be variable and a function of the velocity.

From a series of careful experiments, Colonel Robert has constructed this formula for spherical projectiles, viz:—

Resistance of air $= r = A\pi R^2\left(1 + \frac{v}{a}\right)v^2$, in which A is the resistance, in pounds, on a square foot of a projectile moving with a velocity of 1 foot per second; a is linear and equal to 1427 ft.; $A = .000514$; $\pi R^2 =$ area of the cross section of the ball. For the ball whose diameter is 6 inches

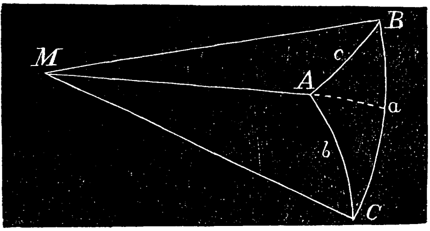
$$r = .000514 \times 3.1416 \times \frac{1}{16} \times \left(\frac{5280}{7}\right)^2 \left(1 + \frac{5280}{9989}\right) = 87 \text{ lbs. nearly.}$$

For the ball whose diameter is $6\sqrt[3]{2}$ in. $r = 138$ lbs. nearly. Consequently there is $138 - 87 = 51$ lbs. more resistance to be overcome. The force necessary to project a body with a given velocity, varies with the density of the body; for, the living force of a body divided by 2 is equal to the inertia of the body overcome in acquiring its velocity = work of the motive force $= \frac{1}{2}mv^2$. $(m = v \times \text{D})$. When D is given, the additional force can easily be calculated. Call it P; then $P +$ the force necessary to overcome a resistance of 51 lbs. shows how much greater force will be required.

54.—"Let ABC represent a spherical triangle and M the centre of the sphere. Find the three distances of the centre of gravity of the spherical triangle ABC from the three planes ABM, ACM and BDM."

SOLUTION BY PROF. J. M. GREENWOOD, KANSAS CITY, MO.

In the diagram M is the center of the sphere; MB, MA, MC, are the three radii; ABC represents the spherical triangle; AMB, AMC, BMC, are the three planes, and a, b, c, the sides opposite the angles.



Put the distances of the three planes from the center of gravity, equal d, d_1, d_2; and let r denote the radius of the sphere, s the area of the spherical triangle, and s_1 the area of its projection on BMC, then

(1) $$d = r\frac{s_1}{s}, \text{ also}$$

(2) $$s = \frac{\pi r^2}{180}\Big(A + B + C - 180\Big).$$

(3) $$s_1 = \text{area } BMC - \text{area } AOB . \cos B - \text{area } AOC . \cos C$$
$$= \frac{\pi r^2}{360}\Big(a - c\cos B - b\cos C\Big).$$

$$\therefore \quad d = \frac{r}{2} . \frac{a - b\cos C - c\cos B}{A + B + C - 180}.$$

In the same way d_1 and d_2 may be found.

55.—"A very small bar of matter is movable about one extremity which is fixed half way between two centers of force attracting inversely as the square of the distance; if l be the length of the bar, and $2a$ the distance between the centers of force, prove that there will be two positions of equilibrium for the bar, or four, according as the ratio of the absolute intensity of the more powerful force to that of the less powerful is or is not greater than $(a + 2l) \div (a - 2l)$: and distinguish between the stable and unstable positions."

SOLUTION BY G. W. HILL.

Assume the fixed extremity of the bar as the origin of coordinates and the direction of the line joining the two centers of force as that of the axis of x. Then x and y being the coordinates of a material point dm of the bar, and X and Y the forces acting on it, we have from the well known equations for the motion of a rigid body

$$S . \frac{xd^2y - yd^2x}{dt}\, dm = S(xY - yX).$$

If M and M' denote the intensity of the force at the unit of distance, we have

$$X = \frac{M(a - x)dm}{[(a - x)^2 + y^2]^{\frac{3}{2}}} - \frac{M'(a + x)dm}{[(a + x)^2 + y^2]^{\frac{3}{2}}},$$
$$Y = -\frac{Mydm}{[(a - x)^2 + y^2]^{\frac{3}{2}}} - \frac{M'ydm}{[(a + x)^2 + y^2]^{\frac{3}{2}}}.$$

Introduce now polar coordinates, and put

$$x = r\cos\theta, \qquad y = r\sin\theta,$$

and since the mass of the bar may be supposed evenly distributed along its length put $dm = dr$, and take the integration with respect to r between the limits 0 and l. These substitutions made in the equation of motion, we get

$$\frac{l^3}{3}\cdot\frac{d^2\theta}{dt^2} = a\sin\theta\int_0^l\left[-\frac{Mrdr}{[a^2-2ar\cos\theta+r^2]^{\frac{3}{2}}}+\frac{M'rdr}{[a^2+2ar\cos\theta+r^2]^{\frac{3}{2}}}\right].$$

Or the integration performed

$$\frac{l^3}{3}\cdot\frac{d^2\theta}{dt^2} = -\frac{M\sin\theta}{[a-l\cos\theta+\sqrt{(a^2-2al\cos\theta+l^2)}]\sqrt{(a^2-2al\cos\theta+l^2)}}$$

$$+\frac{M'\sin\theta}{[a+l\cos\theta+\sqrt{(a^2+2al\cos\theta+l^2)}]\sqrt{(a^2+2al\cos\theta+l^2)}}.$$

This differential equation determines θ and thus the position of the bar at any moment. For equilibrium the right-hand member must vanish, thus $\theta = 0$, $\theta = \pi$ are two positions of equilibrium. If there are any others the equation

$$\frac{[a-l\cos\theta+\sqrt{(a^2-2al\cos\theta+l^2)}]\sqrt{(a^2-2al\cos\theta+l^2)}}{[a+l\cos\theta+\sqrt{(a^2+2al\cos\theta+l^2)}]\sqrt{(a^2+2al\cos\theta+l^2)}} = \frac{M}{M'}$$

must be satisfied. But the numerator of the left member of this equation evidently has its minimum value when $\theta = 0$, and constantly increases from this point until $\theta = \pi$ when the maximum value is attained. On the other hand the denominator has its maximum value when $\theta = 0$, and constantly diminishes from this point until $\theta = \pi$, when the minimum is attained From this it is plain that the minimum value of the left member is $\left(\frac{a-l}{a+l}\right)^2$ and the maximum value $\left(\frac{a+l}{a-l}\right)^2$, and that it continually augments in going from the first to the second.

Hence if $\frac{M}{M'}$ lie between $\left(\frac{a-l}{a+l}\right)^2$ and $\left(\frac{a+l}{a-l}\right)^2$ there will be two additional positions of equilibrium, one between $\theta = 0$ and $\theta = \pi$, and the other between $\theta = \pi$ and $\theta = 2\pi$; in the contrary case there will be none.

When we have nearly $\theta = 0$, the differential equation reduces sensibly to

$$\frac{l^3}{3}\cdot\frac{d^2\theta}{dt^2} = \left[-M(a-l)^{-2}+M'(\mathrm{a}+l)^{-2}\right]\sin\theta,$$

and when nearly $\theta = \pi$, to

$$\frac{l^3}{3}\cdot\frac{d^2\theta}{dt^2} = \left[-M(a+l)^{-2}+M'(a-l)^{-2}\right]\sin\theta$$

Thus the position of equilibrium when $\theta = 0$ is stable or unstable according as $\frac{M}{M'}$ is greater or less than $\left(\frac{a-l}{a+l}\right)^2$, and when $\theta = \pi$, the equilibrium is stable or unstable according as $\frac{M}{M'}$ is less or greater than $\left(\frac{a+l}{a-l}\right)^2$.

The two remaining positions of equilibrium, when they exist, are always unstable as will be plain from considering the mode of increase of the function of θ which is equivalent to $\frac{d^2\theta}{dt^2}$.

Note. The foregoing solution agrees with the statement of the problem, if we suppose that l is so small that its square may be neglected. We may add that the preceding expression for $\frac{d^2\theta}{dt^2}$ is complex only because it is necessary to make $\sin\theta$ appear as a factor. If ϕ and ϕ' denote the angles at the base of the triangle formed by the two centres of force and the extremity of the bar, the differential equation can be written thus

$$\frac{l^3}{3}\cdot\frac{d^2\theta}{dt^2} = -\,2M\sin^2\frac{\phi}{2} + 2M'\sin^2\frac{\phi'}{2}.$$

56.—"Find the real value of $(\sqrt{-1})^{\sqrt{-1}}$."

SOLUTION BY WALTER SIVERLY, OIL CITY, PA.

By Euler's Theorem, $\cos\theta + \sqrt{-1}\,.\sin\theta = e^{\theta\sqrt{-1}}$. Assume $\theta = \frac{1}{2}\pi$. $\sqrt{-1} = e^{\frac{1}{2}\pi\sqrt{-1}}$ $\therefore (\sqrt{-1})^{\sqrt{-1}} = e^{-\frac{1}{2}\pi}$. This Theorem is due to John Bernoulli.

SOLUTION BY HENRY HEATON, DES MOINES, IOWA.

$\text{Log}\,\frac{1+x}{1-x} = 2\left(\frac{x}{1} + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7}\ \&c.\right)$. If $x = \sqrt{-1}$,

$$\log\frac{1+\sqrt{1}}{1-\sqrt{1}} = \log\sqrt{-1} = 2\left(\frac{\sqrt{-1}}{1} - \frac{\sqrt{-1}}{3} + \frac{\sqrt{-1}}{5} - \frac{\sqrt{-1}}{7}\&c.\right).$$

$$\therefore\quad \sqrt{-1}\,\log\sqrt{-1} = 2(-1 + \tfrac{1}{3} - \tfrac{1}{5} + \tfrac{1}{7} - \&c.) = -\,\tfrac{1}{2}\pi.$$

$$\therefore\ \log(\sqrt{-1})^{\sqrt{-1}} = -1.570796 = \overline{2}.429204;\ \therefore (\sqrt{-1})^{\sqrt{-1}} = .2078.$$

57.— "Find the nature of the curve represented by the equation

$$4b^2(x^2+y^2)\,[(x-a)^2+y^2-b^2] - a^2y^2[(x-a)^2+y^2] = 0,$$

and show that when a certain relation exists between a and b, the locus reduces to a right line perpendicular to the axis of x, and a 3rd degree curve which is a trisectrix."

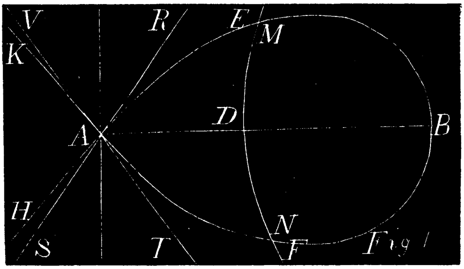
SOLUTION BY E. B. SEITZ, GREENVILLE, OHIO.

Substituting $\rho\cos\theta$ and $\rho\sin\theta$ for x and y, reducing, and solving for ρ, we find

$$\rho = a\cos\theta \pm \frac{2b^2 - a^2\sin^2\theta}{\sqrt{(4b^2 - a^2\sin^2\theta)}}, \;.\;.\;.\;.\;,\;.\;.\;.\;.\;.\;.\;.\;.\;. \quad (1)$$

the polar equation of the curve.

1. When $a > 2b$, the curve consists of two disconnected branches, $HAMBNAK$ and $EMDNF$. (Fig. 1)



When $\theta = 0$, $\rho = a \pm b$; that is, $AB = a + b$, and $AD = a - b$.

When $\sin\theta = b\sqrt{2} \div a$, $\rho = \pm\sqrt{(a^2 - 2b^2)}$, which values determine M and N, the points of intersection of the two branches.

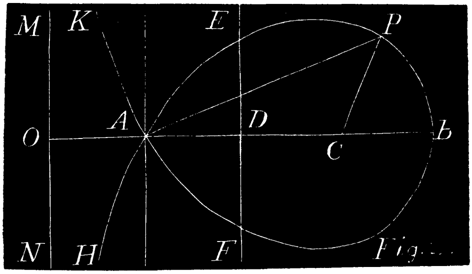
When $\sin\theta = 2b \div a$, $\rho = \infty$; hence each of the branches of the curve has two assymptotes, RS and TV.

2. When $a < 2b$ and $> b\sqrt{2}$, the curve is continuous, the two branches intersecting at the origin, and also at two other points, $\sin\theta = b\sqrt{2} \div a$, $\rho = \pm\sqrt{(a^2 - 2b^2)}$.

3. When $a < b\sqrt{2}$, the curve is continuous, the two branches intersecting at the origin only.

4. When $a = 2b$, equation (1) becomes $\rho = 2b\cos\theta \pm b\left(2\cos\theta - \frac{1}{\cos\theta}\right)$. Separating the second member, we have

$$\rho = \frac{b}{\cos\theta}, \;.\;.\;.\;.\;. \quad (2) \qquad \text{and } \rho = 4b\cos\theta - \frac{b}{\cos\theta}. \;.\;.\;.\;.\;. \quad (3)$$



Equation (2) represents a right line EF, (Fig. 2), perpendicular to the axis of x, at a distance from the origin, $AD = b$. Transforming equation (3) into one in rectangular coordinates, we have

$$y^2 = \frac{x^2(3b - x)}{b + x}.$$

Hence the curve is of the third order. When $\theta = 0$, $\rho = 3b = AB$. When $\theta = 90°$, $\rho = \infty$, and $\rho\cos\theta = -b = AO$. Hence MN is an assymptote.

Lay off $AC = 2b$, and draw AP, CP to any point of the curve. From the equation, we have $\rho\cos\theta = b(4\cos^2\theta - 1)$, or $2\rho\sin\theta\cos\theta = 2b(4\cos^2\theta - 1)\sin\theta$, or $\rho\sin 2\theta = 2b\sin 3\theta$. But from the triangle ACP, we have $\rho\sin P = 2b\sin PCB$. $\therefore$ $\angle P = 2\theta$, and $\angle PCB = 3\theta$; that is, the angle PAB is equal to one-third the angle PCB; hence the name, trisectrix.

58.—"Referring to prob. 33, and Mr. Stille's figure in No. 9, find the rectangular coordinates of the double point not on the axis of x."

SOLUTION BY HENRY GUNDER, GREENVILLE, OHIO.

Drawing tangents to the circle from the double point above the axis of x, and calling the angles made by the radii to the points of tangency, the upper ξ, the lower ξ', and the vectorial angle ψ, from Mr. Stille's (3) we have $\sin 2\xi = \sin 2\xi'$. $\therefore \xi + \xi' = \frac{1}{2}\pi$. Consequently $\rho = \pm r\sqrt{2}$ at the double point. Now $y = \rho \sin \psi = \rho \sin (\frac{1}{4}\pi - \xi') = r[\cos \xi' - \sin \xi']$, . .(1)

$$x = \rho \cos \psi = \rho \cos (\tfrac{1}{4}\pi - \xi') = r[\cos \xi' - \sin \xi'] . . .(2)$$

$$\sin \xi' = \pm \sqrt{\left[\frac{1}{2} \pm \frac{1}{2n}\sqrt{\left(n^2 - 4\right)}\right]} = \frac{1}{2}\left[\mp \sqrt{\left(\frac{n+2}{n}\right)} \pm \sqrt{\left(\frac{n-2}{n}\right)}\right]$$

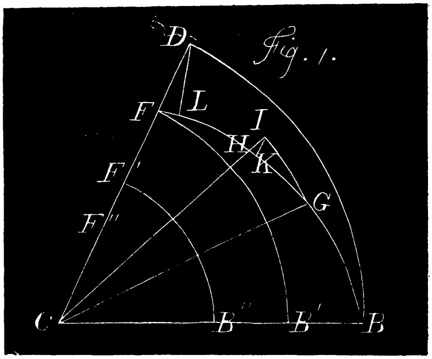
$$\text{and } \cos \xi' = \frac{1}{2}\left[\pm \sqrt{\left(\frac{n+2}{n}\right)} \pm \sqrt{\left(\frac{n-2}{n}\right)}\right].$$

$$\therefore r[\cos \xi' + \sin \xi'] = x = \pm r\sqrt{\left(\frac{n+2}{n}\right)}$$

$$\text{and } r[\cos \xi' - \sin \xi'] = y = \pm r\sqrt{\left(\frac{n-2}{n}\right)}.$$

SOLUTION OF 49, BY R. J. ADCOCK, MONMOUTH, ILL.

"How far will a man travel in unwinding an inch rope from a frustrum of a cone whose upper diameter is 2 ft., lower diameter 15 ft., and height 35 ft., the rope being closely wound around from top to bottom?"



Let the sector CBD be the developed surface of the cone which passes through the central line of the rope, BF, $B'F'$ $B''F''$, &c. the development of that central line, $CG = CI = r =$ the distance from the vertex to any point of the spiral, $IH = dr$, angle $BCG = a$, $GI = rda$, $GH = ds$, $DL = p =$ the constant distance between two coils, $CB = r_1$, $r_2 =$ radius of lower base of the cone on which the central line of the rope is wound. Drop IK perpendicular to GH ($=ds$). Then the arc $BD = 2\pi r_2$ and $2\pi r_1 : 2\pi :: 2\pi r_2 :$ angle $BCD = \pi r_2 \div r_1$, since a point on the spiral ascends along the surface equally for equal angular motion around the axis of the cone,

$$\pi\frac{r_1}{r_2} : p :: da : IK = \frac{pr_2da}{\pi r_1},$$

and by similar elementary triangles, $ds : dr :: rda : IK = \dfrac{rdrda}{ds}$,

$$\frac{rdrda}{ds} = \frac{pr_2da}{\pi r_1},\ ds = \frac{\pi r_1}{pr_2}rdr,\ s = \int_r^{r_1}\frac{\pi r_1}{pr_2}rdr = \frac{\pi r_1}{2pr_2}\left(r^2{}_1 - r\right). \ . \ (1)$$

$$ds^2 = \frac{\pi^2 r_1{}^2}{p^2 r_2{}^2}r^2dr^2 = r^2da^2 + dr^2,\ da = \left(\frac{\pi^2 r_1{}^2}{p^2 r_2{}^2}r^2 - 1\right)^{\frac{1}{2}}r^{-1}dr,$$

$$a = \int_r^{r_1}\left(\frac{\pi^2 r_1{}^2}{p^2 r_2{}^2}r^2 - 1\right)^{\frac{1}{2}}r^{-1}dr = \left(\frac{\pi^2 r_1{}^4}{p^2 r_2{}^2} - 1\right)^{\frac{1}{2}}$$

$$-\tan^{-1}\left(\frac{\pi^2 r_1{}^4}{p^2 r_2{}^2} - 1\right)^{\frac{1}{2}} - \left(\frac{\pi^2 r_1{}^2}{p^2 r_2{}^2}r^2\right)^{\frac{1}{2}} + \tan^{-1}\left(\frac{\pi^2 r_1{}^2}{p^2 r_2{}^2}r^2 - 1\right)^{\frac{1}{2}}. \ . \ . (2)$$

Let D be one position of the extremity of the rope as it is unwound, C the vertex of the cone $CB = r_1$, $BG = r_2$, $DF = s$, the unwound portion, $DG = R$, $\mu = a$ — angle DGB, then $DB^2 = DF^2 - BF^2 = s^2 - (r_1 - r)^2$,

$$\mu = a - \tan^{-1}\frac{\sqrt{[s^2 - (r_1 - r)^2]}}{r_2}. \ . \ . \ . \ (3)$$

$$DG^2 = BD^2 + BG^2 = R^2 = s^2 - (r_1 - r)^2 + r_2{}^2 . (4)$$

Taking the value of r from (4) and substituting this value in (3) after having placed s from (1) in (3) and (4), and a from (2), gives the polar equation of the curve traced by the extremity of the rope, from which by rectification the required distance may be computed.

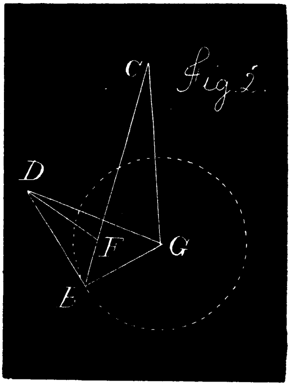


Fig. 2.

PROBLEMS.

59. BY L. REGAN, BOONSBORO, IOWA.—Find a number, consisting of two places of figures, whose half squared will equal the number inverted.

60. BY PHILIP HOGLAN, NEWCOMERSTOWN, O.—Find a value for x which will make the expression

$$\frac{62x + 1984}{x^2 - 1024}$$

a whole number.